A well-balanced numerical scheme for solutions with vacuum to a 1d quasilinear hyperbolic model of chemotaxis

Monika Twarogowska
INRIA Sophia Antipolis - OPALE Project-Team
monika.twarogowska@inria.fr

We consider a hyperbolic system of chemotaxis introduced by Gamba et.al. in [2], which models in vitro experiments of early stages of the vasculogenesis process. It describes the evolution of the density $\rho(x,t)$ of endothelial cells, their velocity $u(x,t)$ and is coupled with a parabolic equation for the concentration $\phi(x,t)$ of a chemical substance. In one space dimension the system writes as

$$\begin{cases} \rho_t + (\rho u)_x = 0, \\ (\rho u)_t + \left(\rho u^2 + P(\rho)\right)_x = -\alpha\rho u + \chi\rho\phi_x, \\ \phi_t = D\phi_{xx} + a\rho - b\phi. \end{cases} \tag{1}$$

Chemoattractant ϕ is released by cells, whereas cells motion is directed by its gradient and is slowed down due to the adhesion with the substratum. Overcrowding of cells is prevented by the pressure law for isentropic gases that is $P(\rho) = \varepsilon\rho^\gamma$, $\gamma > 1$.

This model was developed to describe formation of capillary-like networks from randomly seeded cells. Emerging of such structured patterns corresponds to non constant stationary solutions composed of regions where the density ρ is strictly positive and regions where it vanishes. We first provide a detailed description in the case $\gamma = 2$ of the non constant stationary solutions composed of vacuum and only one interval where $\rho > 0$.

Then we propose a numerical approximation of the chemotaxis system (1) where the hyperbolic part deals with two problems, namely treating vacuum states and having an accurate approximation of the non constant stationary states of the system. We use a scheme that couples a well-balanced strategy, in the framework of the USI method (see [1]), to capture the non constant equilibria for $\gamma > 1$ with an adapted flux solver in order to treat vacuum. Moreover, this scheme preserves the non negativity of the density and shows a small numerical viscosity.

Using this scheme, we study the dependence of the steady states on the length of the domain, the chemosensitivity constant χ, the adiabatic exponent γ, and the initial mass of cells. In particular, we present some cases where the asymptotic state is different from the one obtained using the diffusive parabolic Keller-Segel type model of chemotaxis with a non linear pressure.

References

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[2] Gamba D., Ambrosi D., Coniglio A., de Candia A., Di Talia S., Giraudo E., Serini G., Preziosi L., Bussolino F., Percolation, morphogenesis, and burgers dynamics in blood vessels formation, *Phys Rev Lett*, **90(11)** (2003), pp. 118101

Joint work with: Roberto Natalini (*IAC-CNR*), Magali Ribot (*Université Nice Sophia Antipolis*)