A finite difference/quadrature method for the fractional Laplacian

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A finite difference/quadrature method for the fractional Laplacian is proposed, based on the singular integral representation for the operator. The method combines finite differences with numerical quadrature, to obtain a discrete convolution operator with positive weights. The accuracy of the method is shown to be $O(h^{3-\alpha})$. The treatment of far field boundary conditions using an asymptotic approximation to the integral is used to obtain an accurate method. Numerical experiments on known exact solutions validate the predicted convergence rates. Computational examples include exponentially and algebraically decaying solutions with varying regularity, showing that our method is compared favorably in one dimension to many existing schemes.