# Long time behavior of forced critical SQG

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We address the long-time dynamics of smooth solutions to the forced critical SQG equation

$$\partial_t \theta + \mathcal{R}^\perp \theta \cdot \nabla \theta + \Lambda \theta = f$$

where $f$ is a time-independent force, $\mathcal{R}^\perp = (-\mathcal{R}_2, \mathcal{R}_1)$ are Riesz transforms, and $\Lambda = (-\Delta)^{1/2}$ is the Zygmund operator. Using a nonlinear lower bound for the fractional Laplacian, we give a new proof that the solution instantly becomes Hölder continuous, without the use of DeGiorgi iteration. The nonlinear lower bound for $\Lambda$ also shows that the solution is in fact classical. We prove that for $t \gg 1$ the norm of the solution measured in a sufficiently strong topology depends solely on $f$, and use this fact to show the existence of a global attractor, with finite fractal dimension. Additionally, we prove there is no anomalous dissipation in the long time averages for the viscous perturbations of the system. This is joint work with P. Constantin and A. Tarfulea.