Penalty free Nitsche method for interface problems

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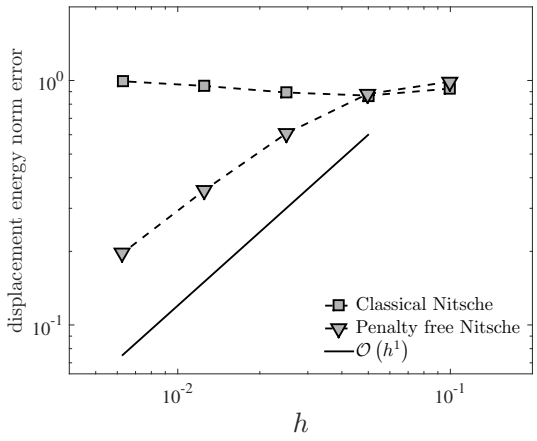
In the finite element method the weak imposition of boundary conditions or interface conditions has become popular over the past few years. Several approaches can be used for this weak enforcement, the Lagrange multiplier method and the Nitsche's method are widely used for this purpose. Nitsche's method is a penalty based method that has a symmetric and a nonsymmetric version. In this work we consider a nonsymmetric penalty free Nitsche's method [4, 2], it can be seen as a Lagrange multiplier method, where the Lagrange multipliers has been replaced by the boundary fluxes of the discrete elliptic operator. This leads to a method that is stable without any unknown parameter and without introducing additional degrees of freedom. The less stiff scheme obtained by eliminating the penalty appears to have some advantages for coupled problems, for instance in multiphysics coupling in continuum mechanics. The following figure is extracted from [3], this paper investigates several time dependent fluid-structure interaction schemes. The graph represented here considers two similar algorithms using a loosely coupled scheme with classical Nitsche and penalty free Nitsche. It shows that in contrast to the classical symmetric Nitsche's method the penalty free algorithm gives a convergent solution.



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The penalty free Nitsche's method is applied to interface problems when the mesh fits the interface (nonconforming domain decomposition [1]) or not (unfitted domain decomposition [1, 5]) for the compressible and incompressible elasticity. Also, we use the method to develop a scheme for unfitted fluid-structure interaction. For each case we show theoretically and numerically the convergence of the error.

References

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