L^p–MAXIMAL REGULARITY ON BANACH SPACES WITH A SCHAUDER BASIS

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Abstract. We investigate the problem of L^p-maximal regularity on Banach spaces having a Schauder basis. Our results improve those of a recent paper. The results contained in this note will be detailled and included in a forthcoming paper.

1. Introduction

We will only recall the basic facts and definitions on maximal regularity. For further information, we refer the reader to [1], [3], [6] or [5].

We consider the following Cauchy problem:

$$\begin{cases} u'(t) + B(u(t)) = f(t) & \text{for } 0 \leq t < T \\ u(0) = 0 \end{cases}$$

where $T \in (0, +\infty)$, $-B$ is the infinitesimal generator of a bounded analytic semigroup on a complex Banach space X and u and f are X-valued functions on $[0, T)$. Suppose $1 < p < \infty$. B is said to satisfy L^p*–maximal regularity* if whenever $f \in L^p([0, T); X)$ then the solution

$$u(t) = \int_0^t e^{-(t-s)B} f(s)\, ds$$

satisfies $u' \in L^p([0, T); X)$. Clearly, this property does not depend on $T \in (0, +\infty)$. Besides, it is known that B has L^p-maximal regularity for some $1 < p < \infty$ if and only if it has L^p-maximal regularity for every $1 < p < \infty$ [2], [3], [7]. We thus say simply that B satisfies *maximal regularity (MR).*

As in [5], we define:

Definition 1.1. A complex Banach space X has the *maximal regularity property* (MRP) if B satisfies (MR) whenever $-B$ is the generator of a bounded analytic semigroup.

Let us recall that De Simon [2] proved that any Hilbert space has (MRP), and that the question whether L^q for $1 < q \neq 2 < \infty$ has (MRP) remained

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open until recently. Indeed, in [5] it is shown that a Banach space with an unconditional basis (or more generally a separable Banach lattice) has (MRP) if and only if it is isomorphic to a Hilbert space.

In this paper we attempt to work without these unconditionality assumptions and study the (MRP) on Banach spaces with a Schauder basis. In particular, we show that a UMD Banach space with a Schauder basis and satisfying (MRP) must be isomorphic to an ℓ_2 sum of finite dimensional spaces.

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2. Notation and background

We will follow the notation of [5]. Let us now introduce more precisely a few notions.

If F is a subset of the Banach space X, we denote by $[F]$ the closed linear span of F.

We denote by $(\varepsilon_k)_{k=1}^\infty$ the standard sequence of Rademacher functions on $[0,1]$.

We denote by $(h_k)_{k\geq 0}$ the Haar system on $L^2([0,1]$ enumerated in its natural order. More precisely: $h_0(t)=1$ on $[0,1]$; for $k\geq 0$ and $0\leq m\leq 2^k-1$ $h_{2^k+m}(t)=1$ if $\frac{m}{2^k}<t\leq\frac{m}{2^k}+\frac{1}{2^{k+1}}$ and $h_{2^k+m}(t)=-1$ if $\frac{m}{2^k}+\frac{1}{2^{k+1}}<t\leq\frac{m+1}{2^k}$.

Let $\alpha_k=\|h_k\|_2^2$.

Let $1\leq p<\infty$. A Banach space X has *type p* if there is a constant $C>0$ such that for every finite sequence $(x_k)_{k=1}^K$ in X:

$$\Big(\int_0^1\|\sum_{k=1}^K\varepsilon_k(t)x_k\|^2\,dt\Big)^{1/2}\leq C\Big(\sum_{k=1}^K\|x_k\|^p\Big)^{1/p}.$$

Notice that every Banach space is of type 1.

A closed subspace E of a Banach space X is said to be *c-complemented* in X if there is a continuous linear projection P from X onto E with $\|P\|\leq c$.

Two Banach spaces X and Y are *c-isomorphic* if there a continuous linear isomorphism T from X onto Y such that $\|T\|\,\|T^{-1}\|\leq c$.

Two basic sequences $(x_k)_{k\geq 1}$ and $(y_k)_{k\geq 1}$ are *c-equivalent* if there is a linear isomorphism T from $[x_k]_{k\geq 1}$ onto $[y_k]_{k\geq 1}$ with $\|T\|\,\|T^{-1}\|\leq c$ and such that for all $k\geq 1$, $T(x_k)=y_k$.

We denote by $\omega^{<\omega}$ the set of all finite sequences of positive integers, including the empty sequence denoted $\emptyset$. For $a=(a_1,..,a_k)\in\omega^{<\omega}$, $|a|=k$ is the *length* of a ($|\emptyset|=0$). For $a=(a_1,..,a_k)$ (respectively $a=\emptyset$), we denote $(a,n)=(a_1,..,a_k,n)$ (respectively $(a,n)=(n)$). A subset β of $\omega^{<\omega}$ is a *branch* of $\omega^{<\omega}$ if there exists $(\sigma_n)_{n=1}^\infty\subset\mathbb{N}$ such that $\beta=\{(\sigma_1,..,\sigma_n);\ n\geq 1\}$.

In this paper, for a Banach space X, we call *tree* in X any family $(y_a)_{a\in\omega^{<\omega}} \subset X$. A tree $(y_a)_{a\in\omega^{<\omega}}$ is *weakly null* if for any $a \in \omega^{<\omega}$, $(y_{(a,n)})_{n\geq 1}$ is a weakly null sequence.
A tree $(y_a)_{a\in\omega^{<\omega}}$ is *a Haar tree* if for any branch β of $\omega^{<\omega}$, $\sum_{a\in\beta} y_a h_a$ converges in $L^2([0,1];X)$.
Let $(y_a)_{a\in\omega^{<\omega}}$ be a tree in the Banach space X. Let $T \subset \omega^{<\omega}$, $(y_a)_{a\in T}$ is a *full subtree* of $(y_a)_{a\in\omega^{<\omega}}$ if $\emptyset \in T$ and for all $a \in T$, there are infinitely many $n \in \mathbb{N}$ such that $(a,n) \in T$. notice that if $(y_a)_{a\in T}$ is a full subtree of a weakly null Haar tree $(y_a)_{a\in\omega^{<\omega}}$, then it can be reindexed as a weakly null Haar tree $(z_a)_{a\in\omega^{<\omega}}$

Let $(E_n)_{n\geq 1}$ be a sequence of closed subspaces of X. Assume that $(E_n)_{n\geq 1}$ is a Schauder decomposition of X and let $(P_n)_{n\geq 1}$ be the associated sequence of projections from X onto E_n. For convenience we will also denote this Schauder decomposition by $(E_n, P_n)_{n\geq 1}$.

We now state a result of [5] that will be an essential tool for this paper:

Theorem 2.1. *Let $(E_n, P_n)_{n\geq 1}$ be a Schauder decomposition of the Banach space X. Let $Z_n = P_n^* X^*$ and $Z = [\cup_{n=1}^{\infty} Z_n]$. Assume X has (MRP). Then there is a constant $C > 0$ so that whenever $(u_n)_{n=1}^N$ are such that $u_n \in [E_{2n-1}, E_{2n}]$ and $(u_n^*)_{n=1}^N$ are such that $u_n^* \in [Z_{2n-1}, Z_{2n}]$ then*

$$\left(\int_0^{2\pi} \|\sum_{n=1}^N P_{2n} u_n e^{i2^n t}\|^2 \frac{dt}{2\pi}\right)^{1/2} \leq C \left(\int_0^{2\pi} \|\sum_{n=1}^N u_n e^{i2^n t}\|^2 \frac{dt}{2\pi}\right)^{1/2}$$

and

$$\left(\int_0^{2\pi} \|\sum_{n=1}^N P_{2n}^* u_n^* e^{i2^n t}\|^2 \frac{dt}{2\pi}\right)^{1/2} \leq C \left(\int_0^{2\pi} \|\sum_{n=1}^N u_n^* e^{i2^n t}\|^2 \frac{dt}{2\pi}\right)^{1/2}.$$

3. The main results

We begin with a general result on spaces with a Schauder basis:

Theorem 3.1. *Let X be a Banach space of type $p > 1$ and with a Schauder basis $(x_n)_{n=1}^{\infty}$. If X has (MRP), then there is a constant $C > 0$ so that for any $(u_k)_{k=1}^K$ block basic sequence with respect to the basis (x_n):*

$$\frac{1}{C}\sum_{k=1}^K \|u_k\|^2 \leq \int_0^1 \|\sum_{k=1}^K \varepsilon_k(t) u_k\|^2\, dt \leq C \sum_{k=1}^K \|u_k\|^2.$$

Proof. We begin with the left hand side inequality. As usual we can replace the ε_k's by their complex valued counterparts: the functions $t \mapsto e^{i2^k t}$. it is then equivalent to show that there exists $C > 0$ so that for any block basic sequence $(u_k)_{k=1}^K$:

$$\int_0^{2\pi} \|\sum_{k=1}^K e^{i2^k t} u_k\|^2 \frac{dt}{2\pi} \geq \frac{1}{C} \sum_{k=1}^K \|u_k\|^2. \qquad (*)$$

Assume that (*) is false. In particular, we can find a block basic sequence $u_1, ..., u_{k_1}$ with $u_k = \sum_{j=r_k}^{r_k+\rho_k} a_j x_j$, $1 = r_1 < .. < r_k < r_k + \rho_k < r_{k+1} < ..$ and

$$\int_0^{2\pi} \|\sum_{k=1}^{k_1} e^{i2^k t} u_k\|^2 \frac{dt}{2\pi} \leq \sum_{k=1}^{k_1} \|u_k\|^2.$$

Now $[x_j,\ j > r_{k_1} + \rho_{k_1}]$ is of type $p > 1$, so by [4] there exist $r_{k_1} + \rho_{k_1} < s_1 < t_1 < \infty$ and a subspace F_1 of $[x_j,\ s_1 \leq j \leq t_1]$ which is 2-complemented in X and 2-isomorphic to $\ell_2^{k_1}$.

Then, still assuming that (*) is false, we can construct by induction a block basic sequence $(u_k)_{k=1}^\infty$ such that

(i) For any $k \geq 1$, $u_k = \sum_{j=r_k}^{r_k+\rho_k} a_j x_j$ with $1 = r_1 < .. < r_k < r_k + \rho_k < r_{k+1} < ..$ and

$0 = k_0 < k_1 < k_2 < .. < k_n < ..$ so that

$$\forall n : \sum_{k=k_n+1}^{k_{n+1}} \|u_k\|^2 \geq 2^n \int_0^{2\pi} \|\sum_{k=k_n+1}^{k_{n+1}} e^{i2^k t} u_k\|^2 \frac{dt}{2\pi}.$$

And, for all n

(ii) $r_{k_n} + \rho_{k_n} < s_n < t_n < r_{k_n+1}$ and a subspace F_n of $[x_j,\ s_n \leq j \leq t_n]$ which is 2-complemented in X and 2-isomorphic to $\ell_2^{k_n - k_{n-1}}$.

Denote now $G_k = [x_{r_k}, .., x_{r_{k+1}-1}]$ if $k \notin \{k_1, .., k_n, ..\}$, $G_{k_n} = [x_{r_{k_n}}, .., x_{s_n-1}]$ and $H_n = [x_{s_n}, .., x_{r_{k_{n+1}}-1}]$. We have blocked the basis (x_n) into the Schauder decomposition:

$$G_1 \oplus ... \oplus G_{k_1} \oplus H_1 \oplus .. \oplus G_{k_n+1} \oplus .. \oplus Gk_{n+1} \oplus H_n \oplus ..$$

In order to create new Schauder decompositions of X, we will need the following elementary lemma, that we state without a proof:

Lemma 3.2. *Let $(E_n)_{n\geq 1}$ be a Schauder decomposition of a Banach space X. Assume that for any $n \geq 1$ there exists a continuous projection P_n from E_n onto a subspace F_{2n} of E_n and denote by F_{2n-1} the kernel of this projection. If the P_n's are uniformly bounded, then $(F_n)_{n\geq 1}$ is also a Schauder decomposition of X.*

By the Hahn-Banach Theorem, there is a norm 1 projection from G_k onto $[u_k]$. We denote by $\tilde{G}_k$ the kernel of this projection and will use the decomposition $G_k = \tilde{G}_k \oplus [u_k]$.

Now, there is a projection of norm at most 2 from H_n onto F_n. Let $\tilde{H}_n$ be its kernel and write $H_n = F_n \oplus \tilde{H}_n$.

Then, by Lemma 3.2,

$$\sum_{n=1}^{\infty} \oplus(\tilde{G}_{k_{n-1}+1} \oplus [u_{k_{n-1}+1}] \oplus .. \oplus \tilde{G}_{k_n} \oplus [u_{k_n}] \oplus F_n \oplus \tilde{H}_n)$$

is a Schauder decomposition of X.

Let us now denote by $(e_k)_{k=k_{n-1}+1}^{k_n}$ a basis of F_n which is 2-equivalent to the canonical basis of $\ell_2^{k_n - k_{n-1}}$. Then

$$\sum_{n=1}^{\infty} \oplus(\tilde{G}_{k_{n-1}+1} \oplus [u_{k_{n-1}+1}] \oplus [e_{k_{n-1}+1}] \oplus .. \oplus \tilde{G}_{k_n} \oplus [u_{k_n}] \oplus [e_{k_n}] \oplus \tilde{H}_n)$$

is also a Schauder decomposition of X.
And by Lemma 3.2, so is

$$\sum_{n=1}^{\infty} \oplus\big((\sum_{k=k_{n-1}+1}^{k_n} \tilde{G}_k \oplus [u_k + \|u_k\| e_k] \oplus [e_k]) \oplus \tilde{H}_n\big).$$

Finally we block this decomposition into a new decomposition $(E_n)_{n\geq 1}$ in such a way that for any $k \geq 1$, $E_{2k} = [e_k]$. Since X has the (MRP), we can apply Theorem 2.1 with $v_k = u_k = u_k + \|u_k\|e_k - \|u_k\|e_k$, for all $k \geq 1$. Then we obtain that there exists $K > 0$ such that:

$$\forall n \geq 1 : \sum_{k=k_n+1}^{k_{n+1}} \|u_k\|^2 \leq K \int_0^{2\pi} \| \sum_{k=k_n+1}^{k_{n+1}} e^{i2^k t} u_k \|^2 \frac{dt}{2\pi}.$$

This is in contradiction with our construction.

If we assume now that the right hand side inequality claimed in Theorem 3.1 is false, we construct similarly a block basic sequence (u_k) denying this inequality in which we interlace some finite dimensional hilbertian spaces. But, with similar notation, we will make use of the Schauder decompositon

$$\sum_{n=1}^{\infty} \oplus\big((\sum_{k=k_{n-1}+1}^{k_n} \tilde{G}_k \oplus [u_k + \|u_k\| e_k] \oplus [u_k]) \oplus \tilde{H}_n\big).$$

Then we block this decomposition into $(E_n)_{n\geq 1}$ in such a way that $E_{2k} = [u_k]$ and apply Theorem 2.1 with $v_k = \|u_k\|e_k = (u_k + \|u_k\|e_k) - u_k$.

□

As an application, we obtain the following result

Theorem 3.3. *Let X be a UMD Banach space with a Schauder basis and satisfying (MRP). Then X is isomorphic to an ℓ_2-sum of finite dimensional spaces.*

Proof. Let $\| \ \|$ be the norm of X, $(x_k)_{k\geq 0}$ be a Schauder basis of X and $(x_k^*)_{k\geq 0}$ the coordinate functionals associated with the basis $(x_k)_{k\geq 0}$. Since X is reflexive, $(x_k^*)_{k\geq 0}$ is a basis of X^*. We will need the applications defined from $\{k\in\mathbb{N};\ k\geq 1\}$ into $\{k\in\mathbb{N};\ k\geq 2\}$ as follows: if $k\leq 0$ and $0\leq m\leq 2^k-1$, we set $\varphi(2^k+m)=2^{k+1}+m$ and $\psi(2^k+m)=2^{k+1}+2^k+m$. We have the following lemma:

Lemma 3.4. *There exists a constant $C>0$ such that for any block basic sequence $(u_k)_{k\geq 0}$ with respect to (x_k), for any $K\geq 0$:*

$$\frac{1}{C}\sum_{k=0}^{K}\alpha_k\|u_k\|^2\leq\int_0^1\|\sum_{k=0}^{K}u_kh_k(t)\|^2\,dt\leq C\sum_{k=0}^{K}\alpha_k\|u_k\|^2.$$

Proof. $(\sum_{k=0}^K u_kh_k)_{K\geq 0}$ is a martingale on $L^2([0,1];X)$. Since X is UMD, there is a constant $C_1>0$ such that for any block basic sequence $(u_k)_{k\geq 0}$ and any $K\geq 0$:

$$\frac{1}{C_1}\int_0^1\int_0^1\|\sum_{k=0}^{K}\varepsilon(s)u_kh_k(t)\|^2\,ds\,dt\leq\int_0^1\|\sum_{k=0}^{K}u_kh_k(t)\|^2\,dt$$

$$\leq C_1\int_0^1\int_0^1\|\sum_{k=0}^{K}\varepsilon(s)u_kh_k(t)\|^2\,ds\,dt.$$

Moreover, since X is UMD, it has a type >1 and the conclusion of the proof of this lemma follows from Theorem 3.1 □

The next step of our proof will be to show that:

Proposition 3.5. *There exist an equivalent norm ρ on X, whose dual norm will be denoted by σ, and a constant $K>0$ so that: $\forall x\in X\ \forall\varepsilon>0\ \exists k_0\in\mathbb{N}$ such that*

$$\forall y\in[x_k,\ k\geq k_0]:\ \rho^2(x+y)\leq\rho^2(x)+K\rho^2(y)+\varepsilon$$

and

$$\forall y^*\in[x_k^*,\ k\geq k_0]:\ \sigma^2(x+y)\leq\sigma^2(x)+K\sigma^2(y)+\varepsilon$$

Proof. Consider the function F defined for $x\in X$ by: $F(x)$ is the infimum of all $\lambda>0$ so that for any weakly null haar tree $(y_a)_{a\in\omega^{<\omega}}$ in X, there is a branch β of $\omega^{<\omega}$ such that:

$$\int_0^1\|x+\sum_{a\in\beta}y_ah_{|a|}(t)\|^2\,dt-C\sum_{a\in\beta}\alpha_{|a|}\|y_a\|^2\leq\lambda,$$

where C is the constant given by Lemma 3.4.
F is clearly continuous and 2-homogeneous.

By a standard diagonal argument, one can show that for any $\lambda > F(x)$ and any weakly null haar tree $(y_a)_{a\in\omega^{<\omega}}$ in X, there is a full subtree $(z_a)_{a\in\omega^{<\omega}}$ of $(y_a)_{a\in\omega^{<\omega}}$ such that for any branch β of $\omega^{<\omega}$:

$$\int_0^1 \|x + \sum_{a\in\beta} z_a h_{|a|}(t)\|^2\, dt - C\sum_{a\in\beta} \alpha_{|a|}\|z_a\|^2 \le \lambda.$$

Using this remark, it is then rather easy to check that F is a convex function. It follows clearly from the definition of F and by considering a null tree that for all x in X, $F(x) \ge \|x\|^2$. On the other hand, let $\lambda < F(x)$, and assume, as we may that x has a finite support with respect to the basis (x_k). Then using an approximation argument, one can find a block basic sequence $(y_k)_{k=0}^{\infty}$, with $y_0 = x$ and

$$\int_0^1 \|x + \sum_{k\ge 1} y_k h_k(t)\|^2\, dt - C\sum_{k\ge 1} \alpha_k\|y_k\|^2 > \lambda.$$

It then follows from Lemma 3.4 that $C\|x\|^2 \ge F(x)$.

Let now $x \in X$, that we will assume again as we may with a finite support with respect to the basis (x_k) and $\varepsilon > 0$, we now claim that there exists $k_1 \ge 1$ such that for any $y \in [x_k,\ k \ge k_1]$:

$$\frac{1}{2}(F(x+y) + F(x-y)) \le F(x) + C\|y\|^2 + \varepsilon.$$

Otherwise, we can pick a weakly null sequence $(y_n)_{n\ge 1}$ so that for all $n \ge 1$:

$$\frac{1}{2}(F(x+y_n) + F(x-y_n)) > F(x) + C\|y_n\|^2 + \varepsilon. \qquad (*)$$

Then, for any $n \ge 1$ there are two weakly null haar trees $(y^+_{n,a})_{a\in\omega^{<\omega}}$ and $(y^-_{n,a})_{a\in\omega^{<\omega}}$ so that for any branch β of $\omega^{<\omega}$:

$$\int_0^1 \|x + y_n + \sum_{b\in\beta} y^+_{n,b} h_{|b|}(t)\|^2\, dt - C\sum_{b\in\beta} \alpha_{|b|}\|y^+_{n,b}\|^2 > F(x+y_n) - \varepsilon$$

and:

$$\int_0^1 \|x - y_n + \sum_{b\in\beta} y^-_{n,b} h_{|b|}(t)\|^2\, dt - C\sum_{b\in\beta} \alpha_{|b|}\|y^-_{n,b}\|^2 > F(x-y_n) - \varepsilon.$$

Then we have, for any branch β of $\omega^{<\omega}$:

$$\frac{1}{2}(F(x+y_n) - F(x-y_n)) - \varepsilon < \int_0^{1/2} \|x + y_n + \sum_{b\in\beta} y^+_{n,b} h_{|b|}(2t)\|^2\, dt$$

$$+ \int_{1/2}^1 \|x - y_n + \sum_{b\in\beta} y^-_{n,b} h_{|b|}(2t-1)\|^2\, dt - C\sum_{b\in\beta} \alpha_{|b|}(\|y^+_{n,b}\|^2 + \|y^-_{n,b}\|^2) =$$

$$\int_0^1 \|x+y_nh_1(t)+\sum_{b\in\beta} y^+_{n,b}h_{\varphi(|b|)}(t)+y^-_{n,b}h_{\psi(|b|)}(t)\|^2\,dt-\frac{C}{2}\sum_{b\in\beta}\alpha_{|b|}(\|y^+_{n,b}\|^2+\|y^-_{n,b}\|^2)$$

Let us now build a new weakly null haar tree $(z_a)_{a\in\omega^{<\omega}}$ as follows: for any $n\in\mathbb{N}$ $z_n=y_n$. If $|a|=\varphi(i)$ with $a=(n,b_1,...,b_k)$, we set $z_a=y^+_{(n,b_1,..,b_i)}$ and if $|a|=\psi(i)$ with $a=(n,b_1,...,b_k)$, we set $z_a=y^-_{(n,b_1,..,b_i)}$.
Using the fact that for any $i\geq 0$, $\alpha_{\varphi(i)}=\alpha_{\psi(i)}=\frac{1}{2}\alpha(i)$, we get that for any $n\geq 1$ and any branch β of $\omega^{<\omega}$:

$$\frac{1}{2}(F(x+y_n)-F(x-y_n))-\varepsilon<\int_0^1\|x+\sum_{b\in\beta}z_bh_{|b|)}(t)\|^2\,dt-C\sum_{b\in\beta,\ |b|\geq 2}\alpha_{|b|}\|z_b\|^2.$$

Then we can extract a branch β of $\omega^{<\omega}$ which is as close as wish to a block basis sequence. Then Lemma 3.4 yields a contradiction with (*).

Now, Since F is convex continuous, we have that for any weakly null sequence (y_n) in X:

$$F(x)\leq\liminf F(x+y_n).$$

So, for any $x\in X$ and any $\varepsilon>0$, there exists $k_2\geq 1$ such that for any $y\in[x_k,\ k\geq k_2]$:

$$F(x+y)\leq F(x)+\|y\|^2+\varepsilon.$$

Setting $\rho(x)=(F(x))^{1/2}$ concludes the proof of the first assertion of Proposition 3.5.

Finally, by using the second inequality in Theorem 2.1 we can prove similarly and simultaneously the second assertion of Proposition 3.5.

□

The end of the proof of Theorem 3.3 relies now on a “skipped blocking” argument.

□

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