Integrating the XXX-Heisenberg spin chain while building conserved charges

Sylvain Labopin

Abstract

In 1919, in his doctoral thesis, J. H. van Leeuwen shown that the magnetism could't be described by the Maxwell's electro-dynamics and the classical statistical mechanics. This phenomenon was explained by Heisenberg and Dirac by using the laws of quantum mechanics. They discovered an effective interaction between electron spins on neighboring atoms with overlapping orbital wave functions. It is caused by the combined effect of the Coulomb repulsion and the Pauli exclusion principle.

Thanks to this discovery, Heisenberg proposed in 1928 a quantum model to describe a linear array of interacting local magnetic moments (representing electron spins with uniform exchange interaction between nearest neighbors) : the $\text{XXX}_{\frac{1}{2}}$ Heisenberg spin chain.

In 1931, Hans Bethe shown its integrability by presenting a method, now called the coordinate Bethe ansatz, for obtaining its exact eigenvalues and eigenvectors.

The aim of this presentation is to recover this ansatz in a more algebraic way.

In order to archive this, we will generalise via the Gell-Mann matrices the $\text{XXX}_{\frac{1}{2}}$ Heisenberg spin chain for spins in superposition of k states and we will construct a family of operators, called T-operators, parametrized by a complex number called the spectral parameter, some others called the homogeneities, an automorphism $g \in GL(\mathbb{C}^k)$ called the twist and an irreducible representation of $GL(\mathbb{C}^k)$, in such a way that each of these operators commutes with the hamiltonian and with the ones which are related to the same twist and the same inhomogeneities.

Thanks to this, we will be able to hope for more information about the spectrum of the theory by studing polynomial relations involving commuting T-operators. Actually, this commutativity implies that these ones have the same eigenspaces than the hamiltonian and then, by considering their restrictions to them, we will obtain relations between number operators which are their eigenvalues, also called the T-functions.

To get such relations, we will use the co-derivative : a way to differentiate families of operators labelled by a twist with respect to it. This process will be very suitable to study the T-operators because it will allow us to express them only in terms of co-derivative, spectral parameter, inhomogeneities and character.